Gradient damage models coupled with plasticity

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It is now well established that gradient damage models are very efficient to account for the behavior of brittle and quasi-brittle materials. Their basic ingredients are: (i) a decreasing dependency of the stiffness $\mathsf{E}(\alpha)$ on the damage variable α; (ii) no more rigidity at the ultimate damage state (say $\mathsf{E}(1) = 0$); (iii) a critical stress σ_c; (iv) a softening behavior with a decrease of the stress from σ_c to 0 when the damage goes to 1; (v) a gradient damage term in the energy which necessarily contains an internal length ℓ and which limits the damage localization. Accordingly, the process of crack nucleation is as follows [4]: (i) a first damage occurs when the stress field reaches the critical stress somewhere in the body; (ii) then, because of the softening character of the material behavior, damage localizes inside a strip the width of which is controlled by the internal length ℓ; (iii) the damage grows inside this strip, but not uniformly in space (the damage is maximal at the center of the strip and is continuously decreasing to 0 so that to match with the undamaged part of the body at the boundary of the strip); (iv) a crack appears at the center of the strip when the damage reaches there its ultimate value (say $\alpha = 1$). During this crack nucleation process, some energy is dissipated inside the damage strip and this dissipated energy involves a quantity $\mathsf{G_c}$ which can be considered as the effective surface energy of Griffith's theory. Therefore, $\mathsf{G_c}$ becomes a byproduct of the gradient damage model which can be expressed in terms of the parameters of the model (specifically, $\mathsf{G_c}$ is proportional to $\sigma_c^2\ell/\mathsf{E}(0)$).

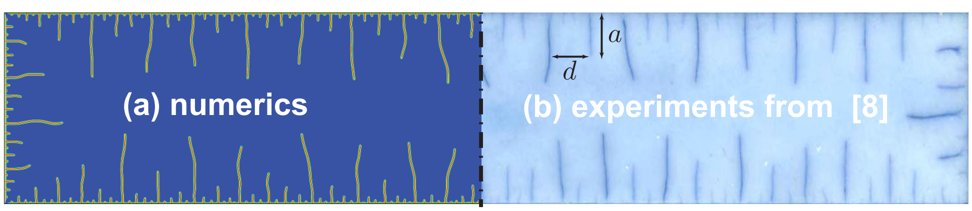

Figure 1: Simulation of a thermal shock in ceramics with a gradient damage model

However, this type of "quasi-brittle" models are not able to account for residual strains and consequently cannot be used in ductile fracture. Moreover there is no discontinuity of the displacement in the damage strip before the loss of rigidity at its center, *i.e.* before the nucleation of a crack. In other words such a model cannot account for the nucleation of cohesive cracks, *i.e.* the existence of surface of discontinuity of the displacement with a non vanishing stress. The natural way to include such effects is to introduce plastic strains into the model and to couple their evolution with damage evolution. To do that we will adopt a variational approach in the spirit of our previous works [2, 3, 5].

Specifically, a variational formulation is proposed for a family of elastic-plastic-damage models within the framework of rate-independent materials [1]. That consists first in defining the total energy which contains, in particular, a gradient damage term and a term which represents the plastic dissipation but depends also on damage. Then, the evolution law is deduced from the principles of irreversibility, stability and energy balance. Accordingly, the plastic dissipation term which appears both in the damage criterion and the plastic yield criterion plays an essential role in the damage-plasticity coupling. Suitable constitutive choices on how the plastic yield stress decreases with damage, allows us to obtain a rich variety of coupled responses. A particular attention is paid on the equations which govern the formation of cohesive cracks where the displacement is discontinuous and the plasticity localizes. In the one-dimensional traction test where the solution is obtained in a closed form, we show that, because of damage localization, a cohesive crack really appears at the center of the damage zone before the rupture and the associated cohesive law is obtained in closed form in terms of the constitutive parameters. A Finite Element discrete version of the energy functional is used to simulate two-dimensional tests; again localized bands of plastic strain can be generated seemingly independent of the mesh size.

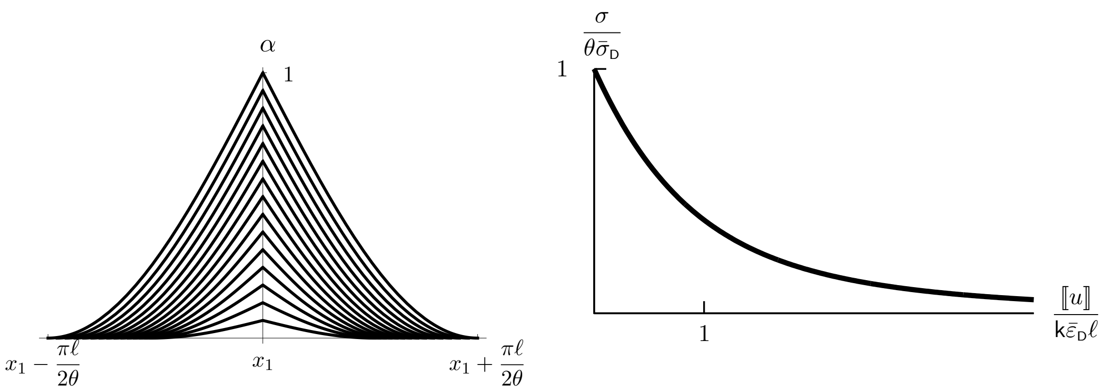

Figure 2: Localization damage process with nucleation of a cohesive crack at the center of the damage zone and the resulting cohesive law

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